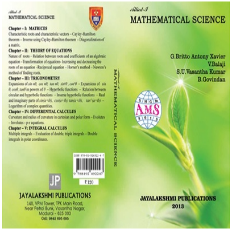
DNMA 15A Basic Mathematics



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Types

1. Exponential Series
2. Logarithmic Series

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For any real number x,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$i.e., \ e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

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Properties and values of e

1. $e^{x+y} = e^{x} e^{y}$
2. $e^{-x} = \dfrac{1}{e^{x}}$
3. $e^{x-y} = \dfrac{e^{x}}{e^{y}}$
4. $e^{rx} = (e^{x})^{r} \quad \forall r$, rational
5. $e^{1} = 2.71828182845...; \quad 2 < e < 3; \quad$ irrational
6. $e^{0} = 1$
7. $e^{\infty} = \infty$
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The nth term of the sequence $a, a+d, a+2d, a+3d, \ldots$ is

$$T_n = a + (n-1)d$$

where **a** - first term and **d** - common difference

$$\frac{Nr}{Dr!} \Rightarrow Nr = A + B(Dr) + C(Dr)(Dr-1) + \ldots$$

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$$1 + \frac{2^3}{2!} + \frac{3^3}{3!} + \frac{4^3}{4!} + \dots$$

Solution:

Let $S = 1 + \frac{2^3}{2!} + \frac{3^3}{3!} + \frac{4^3}{4!} + \dots$

The nth term of the series is $\left(\frac{1^3}{1!}, \frac{2^3}{2!}, \frac{3^3}{3!}, \frac{4^3}{4!}, \dots \right)$

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$$T_n = \frac{n^3}{n(n-1)!}$$

(Since $5! = 5.4.3.2.1 = 5.(4.3.2.1) = 5(4!)$)

$$T_n = \frac{n^2}{(n-1)!}$$

Let $n^2 = A + B(n-1) + C(n-1)(n-2)$

Put $n = 1 \implies A = 1$

$n = 2 \implies A + B = 4$

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$$n = 0 \implies A - B + 2C = 0$$

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Therefore, $n^2 = 1 + 3(n-1) + (n-1)(n-2)$

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$$T_1 = \frac{1}{0!}$$

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Since $S = \sum_{n=1}^{\infty} T_n$

$$S = \left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots\right) + 3\left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \dots\right)$$

$$+ \left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots\right)$$

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Problem 2. Find the sum to infinity of the series

$$\frac{1^2}{3!} + \frac{2^2}{5!} + \frac{3^2}{7!} + \dots$$

Solution:

$$\text{Let } S = \frac{1^2}{3!} + \frac{2^2}{5!} + \frac{3^2}{7!} + \dots$$

The nth term of the series is

$$T_n = \frac{n^2}{(2n+1)!}$$

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Let $n^2 = A + B(2n+1) + C(2n+1)(2n)$

Put $n = -1/2 \implies A = 1/4$

Put $n = 0 \implies A + B = 0$

$\implies 1/4 + B = 0$

$\implies B = -1/4$

put $n = 1 \implies A + 3B + 6C = 1$

$\implies 1/4 - 3/4 + 6C = 1 \implies -1/2 + 6C = 1$

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$$n^2 = 1/4 - (1/4)(2n+1) + (1/4)(2n+1)(2n)$$

$$T_n = \frac{n^2}{(2n+1)!} = \frac{1}{4}\frac{1}{(2n+1)!} - \frac{1}{4}\frac{2n+1}{(2n+1)!} + \frac{1}{4}\frac{(2n+1)(2n)}{(2n+1)!}$$

$$T_n = \frac{1}{4}\frac{1}{(2n+1)!} - \frac{1}{4}\frac{1}{(2n)!} + \frac{1}{4}\frac{1}{(2n-1)!}$$

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$$S = \frac{1}{4}\left(\frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \dots\right) - \frac{1}{4}\left(\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots\right)$$

$$+ \frac{1}{4}\left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots\right)$$

$$S = \frac{1}{4}\left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \dots - 1\right)$$

$$- \frac{1}{4}\left(\frac{1}{0!} + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots - 1\right)$$

$$+ \frac{1}{4}\left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots\right)$$

$$S = \frac{1}{4}\left(\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+\dots\right) - \frac{1}{4}\left(\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\dots\right)$$

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$$S = \frac{1}{4}\left(\frac{e - e^{-1}}{2} - 1\right) - \frac{1}{4}\left(\frac{e + e^{-1}}{2} - 1\right) + \frac{1}{4}\left(\frac{e - e^{-1}}{2}\right)$$

$$S = \frac{1}{4}\left(\frac{e - e^{-1} - 2}{2}\right) - \frac{1}{4}\left(\frac{e + e^{-1} - 2}{2}\right) + \frac{1}{4}\left(\frac{e - e^{-1}}{2}\right)$$

$$S = \left(\frac{e - e^{-1} - 2 - (e + e^{-1} - 2) + e + e^{-1}}{8}\right)$$

$$S = \left(\frac{e - e^{-1}}{8}\right) = \left(\frac{e - \frac{1}{e}}{8}\right)$$

$$\therefore S = \left(\frac{e^2 - 1}{8e}\right)$$

$$S = \frac{1}{4}\left(\frac{e - e^{-1}}{2} - 1\right) - \frac{1}{4}\left(\frac{e + e^{-1}}{2} - 1\right) + \frac{1}{4}\left(\frac{e - e^{-1}}{2}\right)$$

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$$\therefore S = \left(\frac{e^2 - 1}{8e}\right)$$

$$S = \frac{1}{4}\left(\frac{e - e^{-1}}{2} - 1\right) - \frac{1}{4}\left(\frac{e + e^{-1}}{2} - 1\right) + \frac{1}{4}\left(\frac{e - e^{-1}}{2}\right)$$

$$S = \frac{1}{4}\left(\frac{e - e^{-1} - 2}{2}\right) - \frac{1}{4}\left(\frac{e + e^{-1} - 2}{2}\right) + \frac{1}{4}\left(\frac{e - e^{-1}}{2}\right)$$

$$S = \left(\frac{e - e^{-1} - 2 - (e + e^{-1} - 2) + e + e^{-1}}{8}\right)$$

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Problem 3. Find the sum to infinity of the series

$$\frac{2.3}{3!} + \frac{3.5}{4!} + \frac{4.7}{5!} + \dots$$

Solution:

Let $S = \frac{2.3}{3!} + \frac{3.5}{4!} + \frac{4.7}{5!} + \dots$

The nth term of the series is

$$T_n = \frac{(n+1)(2n+1)}{(n+2)!}$$

$$\left(\begin{array}{l} 2, 3, 4 \dots \Longrightarrow t_n = 2 + (n-1) = n+1 \\ 3, 5, 7 \dots \Longrightarrow t_n = 3 + (n-1)2 = 2n+1 \\ 3, 4, 5 \dots \Longrightarrow t_n = 3 + (n-1) = n+2 \end{array} \right)$$

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Let $(n+1)(2n+1) = A + B(n+2) + C(n+2)(n+1)$

Put $n = -2 \implies A = (-1)(-3) \implies A = 3$

Put $n = -1 \implies A + B = 0 \implies B = -3$

Equating the coefficient of n^2, $\implies C = 2$

$\therefore (n+1)(2n+1) = 3 - 3(n+2) + 2(n+2)(n+1)$

$$T_n = \frac{(n+1)(2n+1)}{(n+2)!} = \frac{3}{(n+2)!} - \frac{3(n+2)}{(n+2)!} + \frac{2(n+1)(n+2)}{(n+2)!}$$

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$$T_n = \frac{3}{(n+2)!} - \frac{3}{(n+1)!} + \frac{2}{n!}$$

Put $n = 1, 2, 3, \ldots$

$$T_1 = 3\frac{1}{3!} - 3\frac{1}{2!} + 2\frac{1}{1!}$$

$$T_2 = 3\frac{1}{4!} - 3\frac{1}{3!} + 2\frac{1}{2!}$$

$$T_3 = 3\frac{1}{5!} - 3\frac{1}{4!} + 2\frac{1}{3!}$$

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$$\vdots \qquad \vdots \qquad \vdots$$

$$Since\ S = \sum_{n=1}^{\infty} T_n$$

$$S = 3\left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots - 1 - \frac{1}{1!} - \frac{1}{2!}\right)$$

$$-3\left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots - 1 - \frac{1}{1!}\right)$$

$$+2\left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots - 1\right)$$

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$$S = 3\left(e - 1 - \frac{1}{1!} - \frac{1}{2!}\right) - 3\left(e - 1 - \frac{1}{1!}\right) + 2\,(e - 1)$$

$$S = 3\left(e - 2 - \frac{1}{2!}\right) - 3\,(e - 2) + 2\,(e - 1)$$

$$S = 3\left(\frac{2e - 5}{2}\right) - 3e + 6 + 2e - 2$$

$$S = \left(\frac{6e - 15}{2}\right) - e + 4 = \left(\frac{6e - 15 - 2e + 8}{2}\right)$$

$$S = \left(\frac{4e - 7}{2}\right) = 2e - \frac{7}{2}$$

$$S = 3\left(e - 1 - \frac{1}{1!} - \frac{1}{2!}\right) - 3\left(e - 1 - \frac{1}{1!}\right) + 2(e - 1)$$

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Problem 4. Find the sum to infinity of the series

$$1+\frac{2^4}{2!}+\frac{3^4}{3!}+\frac{4^4}{4!}+\dots$$

Problem 5. Find the sum to infinity of the series

$$\frac{1^2.\,2^2}{1!}+\frac{2^2.\,3^2}{2!}+\frac{3^2.\,4^2}{3!}+\dots$$

Problem 6. Find the sum to infinity of the series

$$5+\frac{2.6}{1!}+\frac{3.7}{2!}+\frac{4.8}{3!}+\dots$$

Problem 7. Find the sum to infinity of the series

$$\frac{1}{1!} + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \dots$$

Problem 8. Find the sum to infinity of the series

$$\frac{1^2}{1!} + \frac{1^2+2^2}{2!} + \frac{1^2+2^2+3^2}{3!} + \dots$$

Problem 9. Sum to infinity of the series

$$\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!}$$

Problem 10. Prove that $\dfrac{e-1}{e+1} = \dfrac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots}{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots}$